

MAS212 Linear Algebra I 2007-2008

Coursework 3

Please hand in your solutions of the **starred** exercises by **2pm on Friday 19 October** using the [blue Linear Algebra I Collection Box](#) on the Ground Floor.

Don't forget to put your **name** and **student number** on your solutions, and to **staple** them.

Exercise 1.

- (a) Use the Invertible Matrix Theorem to show the following: if A and C are square matrices such that $AC = I$, then also $CA = I$; in particular both A and C are invertible with $C = A^{-1}$ and $A = C^{-1}$. [Hint: Show that $Cx = 0$ has only the zero solution.]
- (b) Give an example of a 2×3 matrix A and a 3×2 matrix C such that $AC = I_2$, but $CA \neq I_3$. What is the relevance of this example for the result in (a)?

Exercise* 2. (5 marks)

- (a) Use Gauss-Jordan inversion to determine the inverse of the matrix

$$A = \begin{pmatrix} 1 & 0 & 3 \\ 1 & 1 & 2 \\ 0 & -2 & 3 \end{pmatrix}.$$

- (b) Write the system

$$\begin{array}{rrcr} x_1 & & + & 3x_3 & = & 5 \\ x_1 & + & x_2 & + & 2x_3 & = & 1 \\ & - & 2x_2 & + & 3x_3 & = & 9 \end{array}$$

in matrix form and use the result from (a) to determine its solution set, without performing elementary row operations.

Exercise* 3. (5 marks)

- (a) Find the inverse of the following matrix using Gauss-Jordan inversion and record the elementary row operations performed:

$$A = \begin{pmatrix} 1 & 0 & -4 \\ 5 & 1 & -20 \\ 0 & 0 & 2 \end{pmatrix}.$$

- (b) Write A and A^{-1} as a product of elementary matrices.

Exercise* 4. (2 marks) Verify the Cofactor Expansion Theorem by calculating the determinant of the matrix

$$A = \begin{pmatrix} 1 & 0 & -1 \\ -2 & 7 & 3 \\ -3 & 5 & 4 \end{pmatrix}$$

by expanding across the second row and down the third column.

Exercise* 5. (6 marks) Compute the following determinants. Remember to indicate which method you are using, that is, which row or column operations you carry out or whether you are using a cofactor expansion.

$$(a) \begin{vmatrix} 0 & 1 & 2 & -1 \\ 3 & 4 & 1 & 2 \\ 5 & 3 & 1 & 2 \\ 3 & 4 & 1 & 5 \end{vmatrix} \quad (b) \begin{vmatrix} 1 & 2 & -1 & 3 \\ -1 & 0 & -2 & -5 \\ 3 & 4 & -1 & 15 \\ -2 & 0 & -1 & -20 \end{vmatrix} \quad (c) \begin{vmatrix} 0 & 0 & 0 & -1 & 0 \\ 0 & 2 & 0 & 0 & 0 \\ 0 & 0 & -3 & 0 & 0 \\ 0 & 0 & 0 & 0 & 4 \\ -5 & 0 & 0 & 0 & 0 \end{vmatrix}$$

Exercise* 6. (2 marks) Show that if A is an $n \times n$ matrix and α a scalar then $\det(\alpha A) = \alpha^n \det(A)$. Use this result to show that $\det(A + B) \neq \det(A) + \det(B)$ in general, unless A and B are 1×1 matrices.